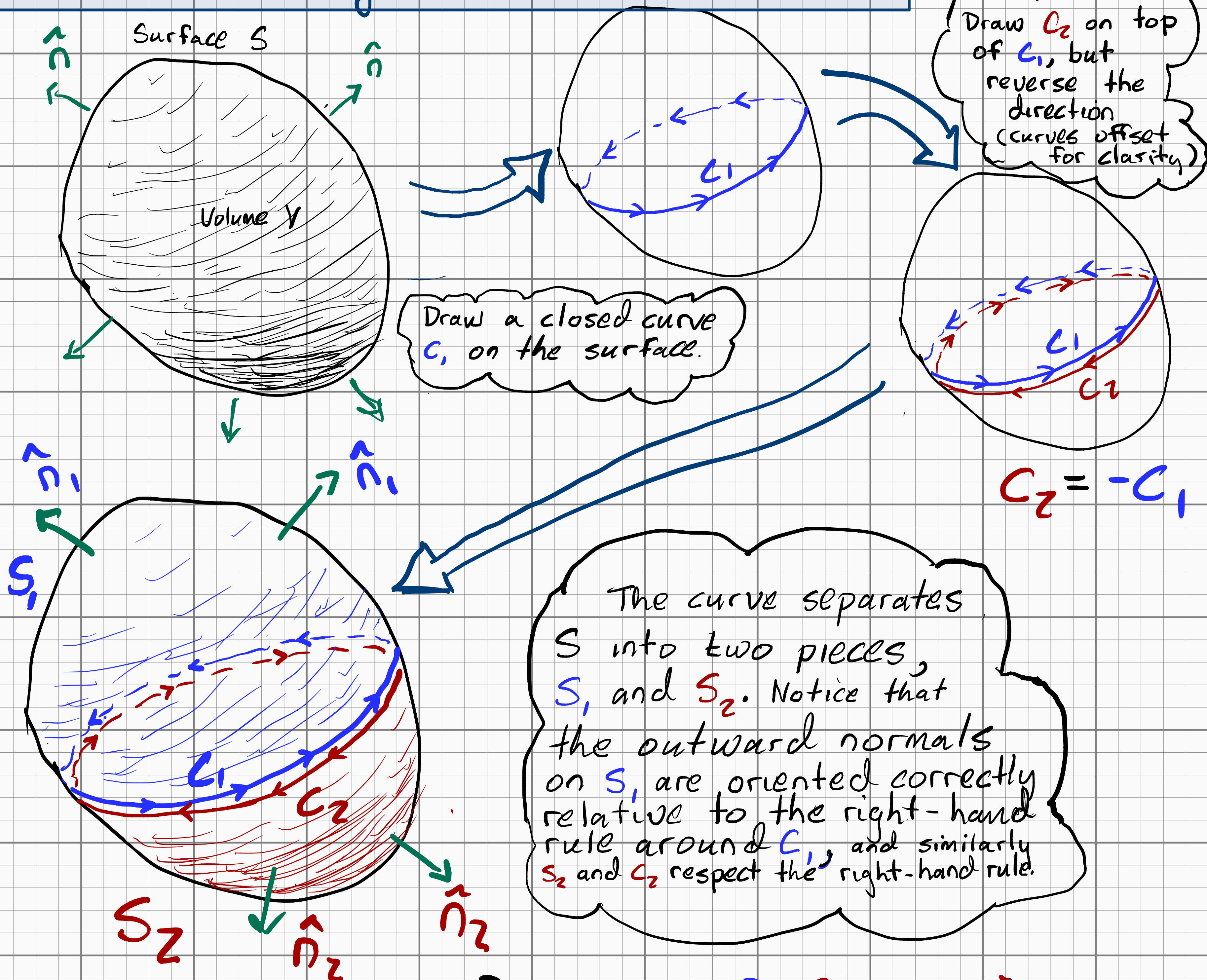


Why must the divergence of a curl be zero?



• Compute $\int_S (\nabla \times \vec{F}) \cdot d\vec{A} = \int_{S_1} \nabla \times \vec{F} \cdot d\vec{A} + \int_{S_2} \nabla \times \vec{F} \cdot d\vec{A}$.

From Stokes' Theorem,

$$\int_{S_1} \nabla \times \vec{F} \cdot d\vec{A} = \int_{C_1} \vec{F} \cdot d\vec{l} \quad \text{and} \quad \int_{S_2} \nabla \times \vec{F} \cdot d\vec{A} = \int_{C_2} \vec{F} \cdot d\vec{l}$$

So then $\int_S (\nabla \times \vec{F}) \cdot d\vec{A} = \int_{C_1} \vec{F} \cdot d\vec{l} + \int_{C_2} \vec{F} \cdot d\vec{l}$ which is zero since $C_2 = -C_1$.

From the Divergence Theorem, $\int_S \vec{G} \cdot d\vec{A} = \int_V \nabla \cdot \vec{G} dV$,
so we have $\int_V \nabla \cdot (\nabla \times \vec{F}) dV = 0$.

Finally, since this equation must hold for every volume V , the integrand must be identically zero:

$$\nabla \cdot (\nabla \times \vec{F}) = 0$$